Neutrino Mass and $\mu \to e + \gamma$ from a Mini-Seesaw

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Abstract

The recently proposed "mini-seesaw mechanism" combines naturally suppressed Dirac and Majorana masses to achieve light Standard Model neutrinos via a low-scale seesaw. A key feature of this approach is the presence of multiple light (order GeV) sterile-neutrinos that mix with the Standard Model. In this work we study the bounds on these light sterile-neutrinos from processes like $\mu \to e + \gamma$, invisible Z-decays, and neutrinoless double beta-decay. We show that viable parameter space exists and that, interestingly, key observables can lie just below current experimental sensitivities. In particular, a motivated region of parameter space predicts a $\mu \to e + \gamma$ branching fraction within the range to be probed by MEG.

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1 Introduction

The discovery of neutrino mass has provided a wealth of information regarding the flavour structure of Nature (for reviews see [1]; for the latest global fit see [2]). However, key questions regarding the neutrino sector remain unanswered and, in particular, the underlying mechanism responsible for neutrino mass remains elusive. The seesaw mechanism provides a simple explanation for the lightness of the known neutrinos [3]. In its standard implementation one assumes $M_R \gg m_{\mathrm{D}} \sim m_W$, where m_{D} is the neutrino Dirac mass, M_R is the singlet Majorana mass and m_W is the W boson mass. The result is a light Standard Model (SM) neutrino with mass $m_\nu \sim m_{\mathrm{D}}^2 / M_R$ and a heavy sterile neutrino with mass $\sim M_R$. Despite the simplicity of this approach, it offers little hope of being experimentally verified due to the invocation of the typically (very) large scale M_R.

A number of alternative mechanisms invoke new physics to explain neutrino masses without relying on a large UV scale. These offer the advantage of being experimentally verifiable and one may hope that the requisite new physics appears at the TeV scale. This can happen in models with radiatively-generated neutrino mass [4], and the mechanism of mass generation can even be connected to the weak scale in models with Coleman-Weinberg symmetry breaking [5]. Of course, Nature is likely not concerned with our ability to experimentally ascertain its inner workings. Nonetheless it is interesting to explore alternative approaches to neutrino mass that may lie within experimental reach; particularly if they enable a natural realization and/or provide insight into the observed flavour structures.

Beyond being SM singlets, right-handed neutrinos can form part of a more complicated hidden sector, and may provide a link to the dark/hidden sector of the Universe. This feature is used to advantage in models that invoke non-standard properties of the gauge-singlet neutrinos in an effort to understand the origin of neutrino mass. For example, interesting exceptions to the standard seesaw picture arise if the right-handed neutrinos are bulk fields in an extra dimension [6] or if they are composite objects within a strongly coupled hidden sector [7].

A recent work has proposed a "mini-seesaw mechanism" in which naturally suppressed Dirac and (sterile) Majorana masses are combined to achieve light SM neutrinos via a low-scale seesaw [8]. This approach borrows elements from both the extra-dimensional scenario and the composite right-handed neutrino approach: the sterile neutrinos are bulk fields in a truncated slice of AdS_5 but, via the AdS/CFT correspondence [9, 10], the model has a dual 4D description in which the right-handed neutrinos are the lightest composites of a strongly coupled hidden sector. A key feature of this approach is the presence of a tower of light (order GeV) sterile neutrinos that can be thought of as either Kaluza-Klein modes or higher fermionic resonances of the CFT (with mass gap). In this work we consider the detailed bounds on these light sterile-neutrinos. We show that viable parameter space exists in which SM neutrino masses can be successfully generated without recourse to large (supra-TeV) scales. Furthermore, we show that it may be possible to observe the effects of the sterile neutrinos in forthcoming experiments: the most stringent constraints come from the FCNC process $\mu \rightarrow e + \gamma$ and, as we will show, the interesting region of parameter space corresponds to $BR(\mu \rightarrow e + \gamma) \sim 10^{-13} - 10^{-12}$. This is right below the current experimental bound and

within the region to be probed by the MEG experiment [11]. The MEG collaboration recently reported that the best value for the number of signal events in their maximal likelihood fit is three [11]. Interestingly, this corresponds to $BR(\mu \to e + \gamma) = 3 \times 10^{-12}$ which, as we show, is obtained via the mini-seesaw with neutrino Yukawa couplings approaching the $\mathcal{O}(0.1)$ level.

Before proceeding we note that bulk gauge-singlet neutrinos in Randall-Sundrum (RS) models [12] were considered in [13] and bulk SM fermions in [14]. Subsequent studies of neutrino mass appeared in [15] and for an incomplete list of recent works in this active field see [16, 17]. Related work on right-handed neutrinos within a strongly coupled CFT was undertaken in [18]. Our implementation within a sub-TeV scale effective theory differs from these previous works. Other works have also considered a low-scale warped hidden sector [19, 20] and for another use of bulk sterile neutrinos see [21]. Light sterile-neutrinos have been studied extensively within the context of the νMSM [22, 23] and for a discussion of the low-scale (GeV) seesaw see [24]. Ref. [25] contains a review on light steriles and a detailed analysis of the bounds on KK neutrinos in ADD models appeared in [26]. A recent work has studied a viable realization of the MEG signal in a SUSY model [27] and other related recent works include [28].

The layout of this paper is as follows. Section 2 describes the model and outlines the basic neutrino spectrum. In Section 3 the sterile neutrino spectrum is considered in more detail and the mixing with the SM is determined. The analysis of the bounds follows. We study the neutrinoless double beta-decay rate and invisible Z-decays in Sections 4 and 5 respectively, a number of other bounds on active-sterile neutrino mixing in Section 6 (e.g. collider constraints), and the lepton number violating decay $\mu \to e + \gamma$ in Section 7. Finally we discuss the prospects for observing invisible Higgs decays in Section 8 and conclude in Section 9.

2 Light Neutrinos from a Mini-Seesaw

We consider a truncated RS model with a warped extra dimension described by the coordinate $z \in [k^{-1}, R]$. A UV brane of characteristic energy scale k is located at $z = k^{-1}$ and an IR brane with characteristic scale R^{-1} is located at $z = R$. The metric is given by

$$ds^2 = \frac{1}{(kz)^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2) = G_{MN}dx^M dx^N, \tag{1}$$

where $M, N, .. (\mu, \nu, ..)$ are the 5D (4D) Lorentz indices and k is the AdS_5 curvature. The characteristic IR scale is suppressed relative to the curvature, as is readily seen using the proper coordinate for the extra dimension.[4] When sourced by a bulk cosmological constant and appropriate brane tensions the metric of Eq. (1) is a solution to the 5D Einstein equations [12]. Furthermore, the length of the space is readily stabilized [29].

We take the SM to be localized on the UV brane, where the natural mass scale is $\sim k$, and accordingly take $k \lesssim M_* \sim$ TeV, where M_* is the cutoff scale. In addition, we consider

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[4]This is defined by $y = k^{-1}\log(kz)$, where $y \in [0, L]$ and $L = k^{-1}\log(kR)$, in terms of which the IR scale is exponentially suppressed, $R^{-1} = e^{-kL}k \ll k$.
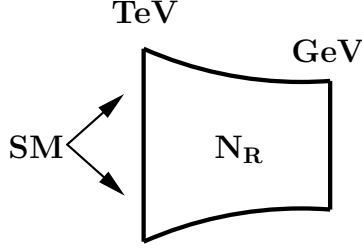
Figure 1: Sketch of the "Little Warped Space". The sterile neutrino N_R propagates in a hidden warped space with order GeV IR scale and order TeV UV scale. Standard Model fields reside on the UV brane and have a suppressed coupling to the chiral zero mode neutrino, which is localized toward the IR brane.

three singlet fermions propagating in the bulk. We label these by N_R as the zero modes will be right-chiral fields that we identify as gauge-singlet neutrinos. The IR scale is nominally taken to be $R^{-1} \sim$ GeV. A sketch of the setup is given in Figure 1. In analogy with the Little RS model [30] we can refer to this as a "Little Warped Space" (LWS). Such a truncated spacetime may seem somewhat unusual at first sight. However, the setup can be thought of as an effective theory that describes the sub-TeV scale physics of a more complete theory, enabling one to consider the effects of a light warped/composite hidden-sector without having to specify the supra-TeV physics. In general the supra-TeV effects will be encoded in UV localized effective operators. As with any sub-TeV scale effective theory, effective operators that break approximate/exact symmetries of the SM must be adequately suppressed to ensure that problematic effects like rapid p-decay and excessive flavour violation do not occur.

The action for a bulk fermion N_R in the background (1) is[5]

$$S_{\text{N}} = \int d^5x \sqrt{G} \left\{ \frac{i}{2} \bar{N}_R \Gamma^B e_B^A \partial_A N_R - \frac{i}{2} (\partial_A \bar{N}_R) \Gamma^B e_B^A N_R + ck \bar{N}_R N_R \right\}, \qquad (2)$$

where $\Gamma^{\mu,5} = \{\gamma^\mu, i\gamma^5\}$ are the 5D Dirac-gamma matrices, e_B^A is the fünfbein and we write the Dirac mass in units of the curvature k. A Kaluza-Klein (KK) decomposition may be performed as

$$N_R(x,z) = (kz)^2 \sum_n \left\{ \nu_L^{(n)}(x) f_L^{(n)}(z) + \nu_R^{(n)}(x) f_R^{(n)}(z) \right\}, \qquad (3)$$

and the bulk wavefunctions $f_{L,R}^{(n)}$ for the chiral components of N_R are readily found [13]:

$$f_{L,R}^{(n)}(z) = \mp \frac{\sqrt{kz}}{N_n} \left\{ J_{\alpha_{\text{L,R}}}(M_n z) + \beta_n Y_{\alpha_{\text{L,R}}}(M_n z) \right\}, \qquad (4)$$

where the order of the Bessel functions is $\alpha_{\text{L,R}} = |c \pm 1/2|$. The boundary conditions force one chirality to be odd and without loss of generality we take this to be the left-chiral field. The single massless mode in the spectrum then has right-chirality.

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[5]We have dropped the spin-connection terms which cancel in S_{N}.

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The KK-expanded Lagrangian is

$$S_{\mathrm{N}} = \sum_n \int d^4x \left\{ i\bar{\nu}^{(n)}\gamma^\mu\partial_\mu\nu^{(n)} - M_n\bar{\nu}^{(n)}\nu^{(n)} \right\}, \tag{5}$$

where $\nu^{(n)} = \nu_L^{(n)} + \nu_R^{(n)}$ is a Dirac fermion with KK mass M_n for $n > 0$ and $\nu^{(0)} = \nu_R^{(0)}$ is the massless right-chiral zero mode, whose bulk profile is

$$f_R^{(0)}(z) = \sqrt{\frac{k(1+2c)}{(kR)^{1+2c}-1}}\,(kz)^c. \tag{6}$$

The dimensionless mass parameter c is seen to control the localization of the zero mode. We will be interested in IR localization with $c \simeq 1$, in order to reduce the wavefunction overlap of the zero mode with UV localized SM neutrinos and suppress the Dirac mass below the weak scale.

The KK masses are determined by enforcing the boundary conditions (Dirichlet/Neumann for $f_{L/R}^{(n)}$), and must satisfy $J_{\alpha_L}(M_n R) \simeq 0$, giving $M_n \simeq (n+c/2)\pi R^{-1}$ for $n > 0$. For $c \simeq 1$ these masses are $M_n R \simeq 4.6,\ 7.7,\ 10.8\ \ldots$ The boundary values of the wavefunctions for the right-chiral modes will be important in the following; for $c \simeq 1$ and $M_n < k$ these are

$$f_R^{(n)}(k^{-1}) \simeq \frac{1}{\Gamma(c+1/2)}\sqrt{\frac{2\pi}{R}}\left(\frac{M_n}{2k}\right)^c,$$

$$f_R^{(n)}(R) \simeq (-1)^n\sqrt{\frac{2}{R}}. \tag{7}$$

At this stage the spectrum consists of a single Weyl neutrino and a tower of Dirac neutrinos with masses $M_n \sim n\pi/R$. We now introduce lepton number violation in the form of a marginal operator on the IR brane:

$$S_{\mathrm{N}} \rightarrow S_{\mathrm{N}} + \frac{\lambda_{\mathrm{N}}}{2}\int d^5x\sqrt{-g_{ir}}\left\{\bar{N}_R^c N_R + \mathrm{H.c.}\right\}\delta(z-R)$$

$$= \sum_{m,n}\int d^4x\left\{i\bar{\nu}^{(n)}\gamma^\mu\partial_\mu\nu^{(n)} - M_n\bar{\nu}^{(n)}\nu^{(n)} + \frac{M_{mn}}{2}(\bar{\nu}_R^{(m)})^c\nu_R^{(n)} + \mathrm{H.c.}\right\}, \tag{8}$$

where the effective Majorana masses are

$$M_{mn} = \lambda_{\mathrm{N}}f_R^{(m)}f_R^{(n)}\Big|_{z=R}. \tag{9}$$

Note that $\nu_L^{(n)}$ does not acquire a boundary Majorana mass as $f_L^{(n)}\big|_R = 0$. For IR localization the zero mode Majorana mass takes a particularly simple form:

$$M_{00} \simeq (1+2c)\frac{\lambda_{\mathrm{N}}}{R}, \tag{10}$$

and the Majorana masses for the $m, n > 0$ modes can be approximately related to that of the zero mode:

$$|M_{0n}| \simeq \sqrt{\frac{2}{2c+1}} M_{00} \quad \text{and} \quad |M_{mn}| \simeq \frac{2M_{00}}{(2c+1)} \quad \text{for} \quad m, n > 0 . \tag{11}$$

We note that $M_{mn} \sim \lambda_{\mathrm{N}}/R$ for all m, n, as expected for an IR localized mass. These Majorana masses mix[6] the KK modes and the true mass eigenstates are linear combinations of $\nu^{(n)}$. As we will detail below, the spectrum consists of a tower of Majorana neutrinos with masses starting at $\sim R^{-1}$. For $\lambda_{\mathrm{N}} \sim 0.1$ one has $M_{00} \sim R^{-1}/10$ and the lightest mode is predominantly composed of $\nu_R^{(0)}$. The higher modes are pseudo-Dirac neutrinos with mass splittings set by $M_{mn} < M_n$. The Dirac mass M_n increases with n as $M_n \sim (n + c/2)\pi/R$ while M_{mn} does not significantly change, so the Majorana masses are increasingly unimportant for the higher modes.

Having outlined the spectrum of sterile neutrinos we can proceed to consider their coupling to the SM. This occurs via a UV localized Yukawa interaction

$$S \supset -\frac{\lambda}{\sqrt{M_*}} \int d^5x \sqrt{-g_{uv}}\ \bar{L}HN_R\ \delta(z - k^{-1}) , \tag{12}$$

where L (H) is the SM lepton (scalar) doublet. After integrating out the extra dimension one obtains Dirac mass terms coupling the KK neutrinos to the SM:

$$S \supset -\sum_n \int d^4x\ m_n^{\mathrm{D}}\ \bar{\nu}_L \nu_R^{(n)} , \tag{13}$$

where $m_n^{\mathrm{D}} = (\lambda \langle H \rangle / \sqrt{M_*}) \times f_R^{(n)}(k^{-1})$ and $\langle H \rangle \simeq 174$ GeV is the vacuum value of the SM scalar. Using the approximations for $f_R^{(n)}(k^{-1})$ with $c \simeq 1$ gives

$$m_0^{\mathrm{D}} \simeq \lambda \sqrt{\frac{k(1+2c)}{M_*}} (kR)^{-c-1/2} \langle H \rangle , \tag{14}$$

$$m_n^{\mathrm{D}} \simeq 2\lambda \sqrt{\frac{2}{M_*R}} \left(\frac{M_n}{2k}\right)^c \langle H \rangle . \tag{15}$$

We will consider the full neutrino mass matrix in more detail and determine the spectrum of physical mass eigenstates in Section 3. Presently, however, we wish to make a few pertinent comments. Including the boundary coupling to SM neutrinos produces a somewhat complicated mass matrix describing both the SM and KK neutrinos. Despite this, the hierarchy of scales generated by the KK profiles allows the basic spectrum to be readily understood. For $n > 0$ the previously pseudo-Dirac neutrinos now have Dirac masses coupling them to the SM, which are roughly $(m_n^{\mathrm{D}}/M_n) \simeq \lambda(\langle H \rangle/k)\sqrt{2/M_*R} \ll 1$ for $c \simeq 1$. This coupling can essentially be treated as a perturbation and to leading order the $n > 0$

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[6]One could instead include the boundary mass in the IR boundary conditions and obtain the full KK spectrum directly [15]. Our main points will be readily seen treating the boundary terms as perturbations.
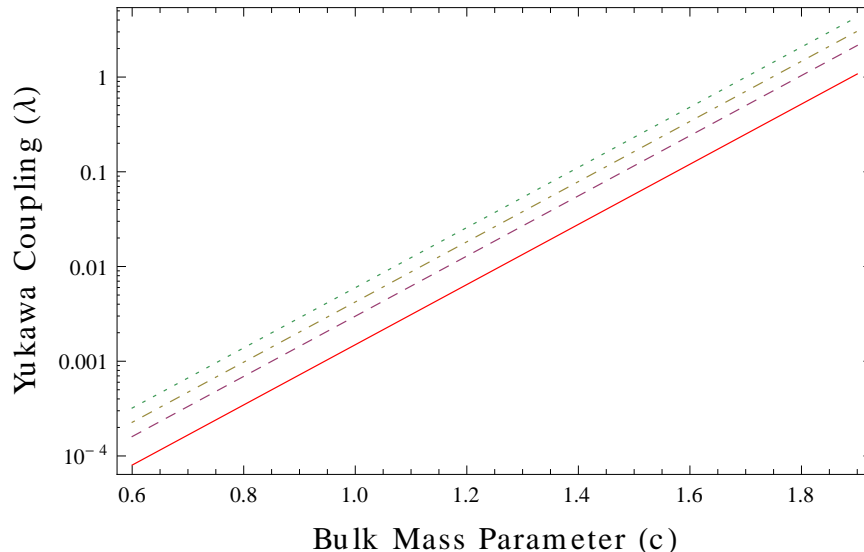
Figure 2: The neutrino Yukawa Coupling (λ) as a function the Bulk Mass Parameter (c) for a fixed value of the SM neutrino mass ($m_\nu = 0.1$ eV). The lines correspond to (bottom to top): $\lambda_{\mathrm{N}} = 0.1,\ 0.4,\ 0.8,\ 1.6$. The IR scale is fixed at $R^{-1} = 1$ GeV and $k = M_*/2 = 1.5$ TeV.

modes remain as pseudo-Dirac neutrinos, comprised predominantly of sterile KK modes. The coupling between the SM and the zero mode is more important: to leading order these two essentially form a standard seesaw pair. The heavy mode has mass of order R^{-1} and is comprised mostly of $\nu^{(0)}$, while the mass of the light SM neutrino is [8]

$$m_\nu \simeq \frac{(m_0^{\mathrm{D}})^2}{M_{00}} \simeq \frac{\lambda^2}{\lambda_{\mathrm{N}}} \frac{\langle H \rangle^2}{M_*} (kR)^{-2c} \ . \tag{16}$$

The key point is that light SM neutrino masses can be obtained with relatively low values of the cutoff, $M_* \sim$ TeV, due to the suppression induced by the extra factor $(kR)^{-2c} \ll 1$. No reliance on large UV scales is necessary.

This approach, referred to as a "mini-seesaw mechanism" [8], generates naturally suppressed SM neutrino masses in a two-fold process. Firstly, the effective 4D Dirac and Majorana mass scales are suppressed: the sub-TeV Majorana scale is generated by warping ($M_{00} \ll m_W$) while the Dirac mass is suppressed by a small wavefunction overlap ($m_0^{\mathrm{D}} \ll M_{00} \ll m_W$). Secondly, a low-scale seesaw operates between the lightest KK mode and the SM neutrino, further suppressing the SM neutrino mass. Together these elements realize the order eV neutrino masses. We emphasize that, unlike most seesaw models with light sterile-Majorana scales, the small neutrino masses are generated naturally and do not require tiny Yukawa couplings *a priori*.

Figure 2 shows a plot of the Yukawa coupling λ as a function of the bulk mass parameter[7] c for the fixed value $m_\nu = 0.1$ eV. It is clear from the plot that m_ν can be readily suppressed

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[7]We restrict our attention to values of $|c|$ no larger than those employed in [30].

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below the weak scale to within the range of interest for the solar and atmospheric neutrino data, even for $\mathcal{O}(1)$ values of the dimensionless couplings λ and λ_N. For future reference we note that $\lambda \in [10^{-2}, 0.1]$ for $c \in [1.3, 1.6]$ when $\lambda_\mathrm{N} = 0.1$.

The goal of this work is to determine the experimental bounds on the tower of low-scale sterile neutrinos. We will focus on values of $R^{-1} \sim$ GeV and investigate whether the bounds allow $\mathcal{O}(1)$ values for the Yukawa coupling λ. Before turning to these matters, we note that, via the application of the AdS/CFT correspondence to RS models [10], the present model is considered dual to a 4D theory in which the UV localized SM fields are treated as fundamental objects external to a hidden strongly-coupled sector. The composite objects correspond to IR localized fields in the 5D picture (roughly; see e.g. [31]) and the lightest fermionic composite is the right-handed neutrino (or neutrinos in a three generation model). Note that IR localization of the Majorana mass term indicates that lepton number is broken within the strongly coupled sector. The dual theory is conformal for energies TeV $\gtrsim E \gtrsim R^{-1}$, with the conformal symmetry broken spontaneously in the IR (dual to the IR brane) and explicitly in the UV (dual to the TeV brane, itself associated with the scale of electroweak symmetry breaking). Thus the Little Warped Space allows one to model light right-handed neutrinos that are composites of a hidden CFT with a sub-weak mass gap.

The small Yukawa coupling between the SM and the lightest right-chiral neutrino is also understood in the dual picture. For every bulk field in the 5D picture, there exists a CFT operator in the dual 4D theory that is sourced by a fundamental field. The source field corresponds to the UV-brane value of the given bulk field in the 5D theory. The SM is external to the CFT, but couples directly to the source field, $N_R|_{z=1/k}$. Writing this field in terms of the physical mass eigenstates introduces the aforementioned tiny mixing angle, the dependence on which ensures the effective Yukawa coupling is highly suppressed.

3 Mixing with a Tower of Sterile Neutrinos

Having specified the model ingredients, we turn to a more detailed consideration of mass mixing in the neutral lepton sector. The neutrino mass Lagrangian can be written as:

$$\mathcal{L}_\nu \supset -\frac{1}{2}\,\bar{\mathcal{V}}^c_{flav}\,\mathcal{M}\,\mathcal{V}_{flav}\ +\ \mathrm{H.c.,} \tag{17}$$

where the neutrino-space flavour basis vector is:

$$\mathcal{V}_{flav}\ =\ (\nu_{\alpha L}, \nu^c_{0R}, \nu^c_{1R}, \nu_{1L}, \nu^c_{2R}, \nu_{2L},\ \dots\ )^T\,, \tag{18}$$

and the subscript "α" denotes that this is a SM flavour eigenstate. The mass matrix is given by

$$\mathcal{M} = \begin{pmatrix} 0 & \mathcal{M}^T_D \\ \mathcal{M}_D & \mathcal{M}_H \end{pmatrix}\,, \tag{19}$$

where the Dirac mass matrix is

$$\mathcal{M}_D = (\ m^\mathrm{D}_0,\ m^\mathrm{D}_1,\ 0,\ m^\mathrm{D}_2,\ 0,\ \dots)^T\,, \tag{20}$$

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and the mass matrix for the "heavy" KK modes is

$$\mathcal{M}_H = \begin{pmatrix} -M_{00} & -M_{01} & 0 & -M_{02} & 0 & \cdots \\ -M_{01} & -M_{11} & M_1 & -M_{12} & 0 & \cdots \\ 0 & M_1 & 0 & 0 & 0 & \cdots \\ -M_{02} & -M_{12} & 0 & -M_{22} & M_2 & \cdots \\ 0 & 0 & 0 & M_2 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} . \tag{21}$$

The Dirac masses m_n^{D} and the Majorana masses $M_{nn'}$ were defined in the previous section. A mass matrix with this form was discussed recently in Ref. [32] and an exact expression for the inverse was obtained:

$$\mathcal{M}_H^{-1} = \begin{pmatrix} \frac{-1}{M_{00}} & 0 & \frac{M_{01}}{M_{00}} \frac{-1}{M_1} & 0 & \frac{M_{02}}{M_{00}} \frac{-1}{M_2} & \cdots \\ 0 & 0 & \frac{1}{M_1} & 0 & 0 & \cdots \\ \frac{M_{01}}{M_{00}} \frac{-1}{M_1} & \frac{1}{M_1} & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & \frac{1}{M_2} & \cdots \\ \frac{M_{02}}{M_{00}} \frac{-1}{M_2} & 0 & 0 & \frac{1}{M_2} & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} . \tag{22}$$

We will use this result below. Note that, in deriving the above expression for the inverse matrix, one makes use of relations between the elements $M_{nn'}$ which result from the underlying warped space derivation. Though the context in which this matrix was considered in [32] differs from that considered here, the same relations amongst the elements $M_{nn'}$ are present in our case. The one generation result presented in Ref. [32] also generalizes to three generations; i.e. the elements of the matrix presented in [32] (and here) can be treated as matrices for the three generation case [33]. We focus on the one generation case but keep some expressions general.

Diagonalization of the matrix $\mathcal{M}$ proceeds as follows. First we perform a standard seesaw rotation by defining the matrix $\mathcal{U}$ as

$$\mathcal{U} \simeq \begin{pmatrix} 1 & \mathcal{M}_D^T \mathcal{M}_H^{-1} \\ -(\mathcal{M}_D^T \mathcal{M}_H^{-1})^T & 1 \end{pmatrix}, \tag{23}$$

giving

$$\mathcal{U}^T \mathcal{M} \, \mathcal{U} \simeq \begin{pmatrix} -\mathcal{M}_D^T \mathcal{M}_H^{-1} \mathcal{M}_D & 0 \\ 0 & \mathcal{M}_H \end{pmatrix}, \tag{24}$$

in the basis $\mathcal{U}^\dagger \mathcal{V}_{flav}$. To diagonalize the heavy KK matrix we write it as $\mathcal{M}_H = \mathcal{M}_{KK} + \mathcal{M}'$, where the first (second) term contains the KK Dirac (Majorana) masses M_n $(M_{nn'})$. We treat the Majorana masses as a perturbation on the matrix $\mathcal{M}_{KK}$. As shown in the previous section, the elements $M_{nn'}$ are all $\mathcal{O}(\lambda_{\mathrm{N}}/R)$, while the KK masses grow like $M_n \sim n\pi/R$. Thus for $|\lambda_{\mathrm{N}}| \lesssim 1$ one has $|M_{nn'}| \ll M_n$ for $n \gg 1$, and the Majorana masses merely perturb the spectrum of the higher KK modes. For the modes with $n \sim 1$ one still has $|M_{nn'}| < M_n$ for $|\lambda_{\mathrm{N}}| \lesssim 1$, but the Majorana masses can be more important as λ_{N} approaches and exceeds

unity from below. For $\lambda_N \lesssim 10^{-1}$ it is valid to treat the Majorana masses as perturbations for the entire tower. For simplicity we focus on this region of parameter space but mention some differences where important.

Next we define the matrix $\tilde{\mathcal{U}}$ as[8]

$$\tilde{\mathcal{U}} = \mathrm{diag}\,(1, U_{KK})\,, \tag{25}$$

where U_{KK} diagonalizes the KK matrix $\mathcal{M}_H$:

$$U_{KK}^T \mathcal{M}_H\, U_{KK} = \mathrm{diag}(\ M_0,\ M_{+1},\ M_{-1},\ M_{+2},\ M_{-2},\ \ldots)\,, \tag{26}$$

and the leading order expression for U_{KK} can be found in Appendix A. The mass eigenvalues are

$$
\begin{aligned}
m_\nu &\simeq \frac{(m_0^D)^2}{M_{00}} \simeq \frac{\lambda^2}{\lambda_N} \frac{\langle H \rangle^2}{M_*} (kR)^{-2c} \\
M_0 &\simeq -M_{00} \\
M_{\pm n} &\simeq \pm M_n - \frac{1}{2} M_{nn},
\end{aligned} \tag{27}
$$

where we include the light mass eigenvalue m_ν for completeness. Negative mass eigenvalues can be brought positive by an appropriate field redefinition, as per usual. We will leave the negative signs so we don't need to keep track of phases in the mixing matrix. Let us emphasize that the above expressions for the $n > 0$ KK masses are correct to $\mathcal{O}(M_{nn'}/M_n)$; their precise form can change if $M_{nn'}$ is increased such that $M_{nn'} \gtrsim M_n$ for some values of n. However, the expression for the SM neutrino mass, $m_\nu \simeq (m_0^D)^2/M_{00}$, is exact to $\mathcal{O}(\mathcal{M}_D^T \mathcal{M}_H^{-1})$ and does not depend on the relation between $M_{nn'}$ and M_n. It is interesting that the leading order seesaw-suppressed mass depends only on the Majorana mass of the zero-mode.

The flavour basis can be expressed in terms of the mass basis:

$$
\begin{aligned}
\mathcal{V}_{flav} &= \mathcal{U} \tilde{\mathcal{U}} \mathcal{V}_{mass} \\
&\simeq \begin{pmatrix} 1 & (\mathcal{M}_D^T \mathcal{M}_H^{-1}) U_{KK} \\ -(\mathcal{M}_D^T \mathcal{M}_H^{-1})^T & U_{KK} \end{pmatrix} \mathcal{V}_{mass}\,,
\end{aligned} \tag{28}
$$

where the mass eigenstates are written as

$$\mathcal{V}_{mass} = (\nu_{iL},\, N_{0R}^c,\, N_{+1L},\, N_{-1L},\, N_{+2L},\, N_{-2L},\, \ldots\,)^T\,, \tag{29}$$

and the subscript "i" denotes a (mostly) SM mass eigenstate. We can write the SM flavour eigenstate in terms of the mass eigenstates as

$$\nu_{\alpha L} = \nu_{iL} + \theta_{\alpha 0} N_{0R}^c + \sum_{n>0} (\theta_{\alpha,+n} N_{+n} + \theta_{\alpha,-n} N_{-n})_L\,, \tag{30}$$

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[8]In a three generation analysis the entry "1" in (25) is replaced by the PMNS mixing matrix U_ν, which diagonalizes the light-sector matrix $\mathcal{M}_\nu = -\mathcal{M}_D^T \mathcal{M}_H^{-1} \mathcal{M}_D$.

where $\theta_{\alpha,\pm n}$ is the mixing matrix element between ν_α and the $\pm n$th level KK mode. For much of what follows we will not need to differentiate the SM flavour eigenstates and accordingly refer to $\theta_{\alpha,\pm n}$ simply as $\theta_{\pm n}$. In the limit of a vanishing boundary Majorana mass, $\lambda_N \to 0$, the two modes $N_{\pm n}$ fuse together to form a single Dirac neutrino with mass M_n, and one has $N_{\pm nL} = (\nu_{nR}^c \pm \nu_{nL})/\sqrt{2}$.

Treating the boundary Majorana mass terms as a perturbation on the KK mass matrix, the mixing angles are approximately

$$
\begin{aligned}
\theta_0 &\simeq -\frac{m_0^D}{M_{00}} + \sum_{n>0} \frac{M_{0n}}{M_n} \frac{m_n^D}{M_n} \\
\theta_{\pm n} &\simeq \frac{1}{\sqrt{2}} \frac{m_n^D}{M_n} \left(\pm 1 + \frac{M_{nn}}{4M_n} \right) + \sum_{n' \neq n} \frac{m_{n'}^D}{\sqrt{2}} \left(\frac{M_{nn'}}{M_{n'}^2 - M_n^2} \right).
\end{aligned}
$$

We will make use of these expressions in what follows. Note that the n-dependence of the Majorana mass appearing in the sum term of θ_0 is $M_{0n} \sim (-1)^n/R$. Consequently the sum piece in the expression for θ_0 is found to be subdominant for $\lambda_N \lesssim 0.1$, giving $\theta_0 \simeq -m_0^D/M_{00}$, which is of the usual seesaw form. Similarly, the sum piece in the expressions for $\theta_{\pm n}$ gives a sub-leading contribution.

Before turning to a detailed analysis of the bounds, we note that the hidden sector also contains a tower of KK gravitons with mass splittings of order $R^{-1} \sim$ GeV. In the present work we follow Ref. [30] and give the bulk graviton a Dirichlet BC on the UV brane. This projects the zero-mode graviton out of the spectrum and ensures that the higher modes do not couple directly to the UV-localized SM. In this case 4D gravity is not included and the model deals purely with particle physics. The spectrum also contains a physical 4D scalar (the radion) that can couple to the UV-localized SM. However, with the standard parametrization of the radion fluctuation [34] we find that the radion $r(x)$ couples to the SM fields like $r(x)/\Lambda_r$, with the dimensionful parameter being $\Lambda_r \sim (kR) \times M_* \gg M_*$. Radion interactions can thus be safely neglected for $M_* \sim$ TeV. In a more complete treatment, one could aim to retain 4D gravity by including a large UV-localized Einstein-Hilbert term and analyzing the metric fluctuations in the interval approach to braneworld gravity [35]. The KK gravitons would be largely repelled from the UV brane so their coupling to UV-localized SM fields remains sufficiently weak to avoid experimental constraints. The radion coupling may also be modified by this procedure. We hope to study these matters in a future work.

4 Neutrinoless Double Beta-Decay

With the above information we have all we need to study the bounds on the mini-seesaw. The sterile KK modes provide additional channels for neutrinoless double beta-decay ($0\nu\beta\beta$-decay), the non-observation of which provides bounds on the parameter space. These bounds can be approximated by noting that the amplitude for $0\nu\beta\beta$-decay from SM neutrinos goes

like [36]

$$\mathcal{A} \propto G_F^2 \frac{\langle m_{ee} \rangle}{q^2}, \tag{31}$$

where $\langle m_{ee} \rangle = \sum_i |U_{ei}^2 m_i|$, G_F is the Fermi constant, and $q^2 \simeq (0.1 \text{ GeV})^2$ is the typical momentum exchanged during the process. The amplitude for $0\nu\beta\beta$-decay induced by a single sterile Majorana neutrino (N_s) that mixes with the SM neutrino via $\nu_\alpha \simeq \nu_i + \theta_s N_s$ goes like

$$\mathcal{A}_s \propto G_F^2 \frac{\theta_s^2}{M_s}. \tag{32}$$

Using the bound $\langle m_{ee} \rangle \lesssim 0.5$ eV and assuming that the sterile neutrino contribution dominates, one obtains the bound $\theta_s^2/M_s \lesssim 5 \times 10^{-8}$ GeV^{-1} [36, 37].

We can generalize this result for a tower of sterile neutrinos. The $0\nu\beta\beta$-decay amplitude produced by the tower is

$$\mathcal{A}_{KK} \propto G_F^2 \frac{\theta_0^2}{M_0} + \sum_{n>0} G_F^2 \left\{ \frac{\theta_{-n}^2}{M_{-n}} + \frac{\theta_{+n}^2}{M_{+n}} \right\}, \tag{33}$$

and one therefore requires

$$3 \times \left| \frac{\theta_0^2}{M_0} + \sum_{n>0} \left\{ \frac{\theta_{-n}^2}{M_{-n}} + \frac{\theta_{+n}^2}{M_{+n}} \right\} \right| \lesssim 5 \times 10^{-8} \text{ GeV}^{-1}, \tag{34}$$

where we make use of the fact that $M_n^2 \gg q^2$ for the parameter space we consider. We also include a sum over three generations of sterile neutrinos, assuming that the active/sterile mixing is similar for each generation (i.e. we do not assume any tuned cancellations of leading terms [24, 23]). With the information from the previous sections one can calculate the the above factor and determine the size of the $0\nu\beta\beta$-decay amplitude. In general the resulting expressions are rather complicated and numerical evaluation is required. However, some simple leading-order expressions can be obtained for the regions of parameter space considered here. Approximating the mixing angles by their leading terms:

$$\theta_0 \simeq -\frac{m_0^{\text{D}}}{M_{00}},$$

$$\theta_{\pm n} \simeq \frac{1}{\sqrt{2}} \frac{m_n^{\text{D}}}{M_n} \left(\pm 1 + \frac{M_{nn}}{4M_n} \right),$$

and noting that

$$\sum_{n>0} \left\{ \frac{\theta_{-n}^2}{M_{-n}} + \frac{\theta_{+n}^2}{M_{+n}} \right\} \simeq \sum_{n>0} \frac{1}{M_n} \left\{ \theta_{+n}^2 - \theta_{-n}^2 + \frac{M_{nn}}{2M_n}(\theta_{+n}^2 + \theta_{-n}^2) + \dots \right\}, \tag{35}$$

one has

$$\sum_{n>0} \left\{ \frac{\theta_{-n}^2}{M_{-n}} + \frac{\theta_{+n}^2}{M_{+n}} \right\} \simeq \sum_{n>0} \left(\frac{m_n^{\text{D}}}{M_n} \right)^2 \left(\frac{M_{nn}}{M_n} \right) \frac{1}{M_n} + \dots. \tag{36}$$
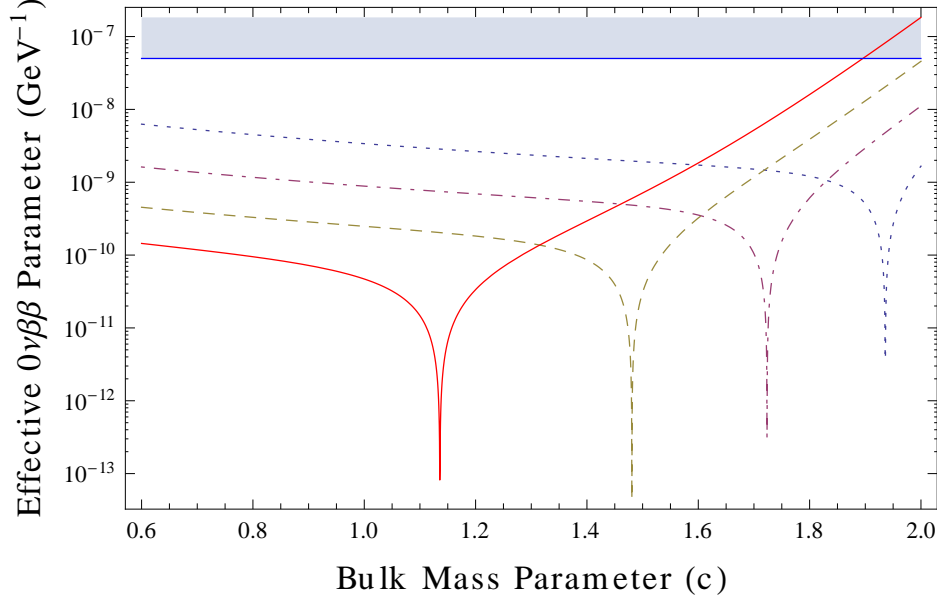
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Figure 3: The Effective Neutrinoless Double Beta-Decay Parameter $(\sum_n \theta_n^2/M_n)$ as a function of the Bulk Mass Parameter (c). The SM neutrino masses are fixed at $m_\nu = 0.1$ eV and the dotted (dot-dashed, dashed, solid) line corresponds to $\lambda_{\rm N} = 0.1$ (0.2, 0.4, 0.8). The shaded region is experimentally excluded. We use $R^{-1} = 1$ GeV and $k = M_*/2 = 1.5$ TeV for the plot.

Note that the nth term is proportional to the lepton number violating mass M_{nn}; in the limit that the Majorana mass vanishes the contributions to $0\nu\beta\beta$-decay from the modes $\pm n$ cancel each other out. For c in the vicinity of unity the above may be expressed in terms of the light SM neutrino mass:[9]

$$3 \times \left[\frac{\theta_0^2}{M_0} + \sum_{n>0} \left\{ \frac{\theta_{-n}^2}{M_{-n}} + \frac{\theta_{+n}^2}{M_{+n}} \right\} \right] \tag{37}$$

$$\simeq \left(\frac{m_\nu}{10^{-1}~{\rm eV}} \right) \left(\frac{10^{-1}}{\lambda_{\rm N}} \right)^2 \left(\frac{\rm GeV}{R^{-1}} \right)^2 \times \left| \frac{-3}{(1+2c)^2} + 3\lambda_{\rm N}^4 \sum_{n>0} \left(\frac{M_n R}{2} \right)^{2(c-2)} \right| \times 10^{-8}~{\rm GeV}^{-1}.$$

The above expression can be evaluated numerically, with the sum over KK modes cutoff in the UV at $n_* \simeq M_* R/\pi$. In practise, no significant error is introduced if one takes the RS2 limit and replaces the sum by a definite integral: $\sum_{n=1}^{n_*} \to \int_1^{n_*} dn$.

We plot Eq. (37) as a function of the bulk mass parameter c for a fixed value of the SM neutrino mass ($m_\nu = 0.1$ eV) in Figure 3. For completeness we include the sub-leading terms in the mixing angles for the plot, though the result is well approximated by the above expression. The experimental bound is included in the plot for comparison and we take $R^{-1} = 1$ GeV, $k = 1.5$ TeV and $k/M_* = 1/2$. Observe that the effective $0\nu\beta\beta$-decay

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[9]We have taken $\lambda_{\rm N} > 0$ in Eq. (37). More generally, the factor of "-1" is replaced by $(-1) \times {\rm sgn}(\lambda_{\rm N})$.

parameter is below the experimental bound for most of the parameter space: all but the largest values of c are viable, with some dependence on λ_{N}.

It is instructive to comment on the λ_{N} dependence in the above expressions (and the figure). For small values of λ_{N} the amplitude for $0\nu\beta\beta$-decay is dominated by the zero-mode contribution. This is because the KK neutrinos $N_{\pm n}$ form a pseudo-Dirac pair for small λ_{N} and the $0\nu\beta\beta$-decay contributions from the two modes tend to cancel each other out. The zero mode has no pseudo-Dirac partner and for small λ_{N} its mass is the only one that "badly" breaks lepton number symmetry; thus its contribution to the lepton-number violating $0\nu\beta\beta$-decay rate is dominant. For small values of λ_{N} the overall $0\nu\beta\beta$-decay rate diminishes with increasing λ_{N} due to the increase in the zero-mode mass. This behaviour is observed in the small c regions in Figure 3.

On the other hand, for larger values of λ_{N} the contribution from the KK modes is seen to dominate. The mass splitting between the pseudo-Dirac KK pairs increases for increasing λ_{N}, $|M_{-n}| - |M_{+n}| \sim \lambda_{\mathrm{N}}/R$. The KK modes $N_{\pm n}$ thus become less pseudo-Dirac-like and form a standard pair of Majorana particles. Accordingly, lepton-number symmetry is more severely broken in the higher KK sector and the $0\nu\beta\beta$-decay signal increases. The cusps in Figure 3 occur at the transition between zero-mode dominance and KK mode dominance; one observes that, once the KK contribution is dominant, the overall rate increases with λ_{N} for a given value of c.

For perturbative values of λ_{N}, only the low-lying modes ($n \lesssim 10$) are expected to have sizeable mass splittings and thus contribute significantly to $0\nu\beta\beta$-decay. The relative splitting goes like $1/n$, $(|M_{-n}| - |M_{+n}|)/M_n \sim \lambda_{\mathrm{N}}/n\pi$, and the decreased splitting suppresses the contribution of the higher modes.

We have considered the KK neutrino contribution to $0\nu\beta\beta$-decay in this section. There is also a contribution from the SM neutrinos which, in general, will interfere with the zero-mode contribution (these two form a standard seesaw pair and thus possess opposite CP parities). However, the contributions from the $n > 0$ modes and the SM neutrinos can add constructively, so the total rate can be larger if c is such that the KK contribution dominates the zero-mode. Given the potential experimental reach for an inverted mass hierarchy [38], the additional contribution from the KK neutrinos can improve the prospects for $0\nu\beta\beta$-decay observation. Let us emphasize, however, that the SM contribution will typically dominate the KK contribution in the region of parameter space where the largest KK contribution comes from the zero-mode. It has been argued that, generically, in seesaw models the contribution from the SM neutrinos is larger than the sterile neutrino contribution [39]. This holds in the present case for the $0\nu\beta\beta$-decay contributions from the zero-mode and the SM neutrino, which essentially form a standard seesaw pair. Indeed, for $\langle m_{ee} \rangle \sim m_\nu = 0.1$ eV one obtains a contribution of $\langle m_{ee} \rangle / q^2 \sim 10^{-8}$ GeV^{-1} to the quantity plotted in Figure 3, which always dominates the zero-mode contribution, but can be exceeded by the sum over higher KK modes for larger c. We only plot the KK contribution in Figure 3 to emphasize the behaviour of this new contribution.

5　Invisible Z-Decays

Mixing between the SM neutrinos and sterile neutrinos induces an effective coupling between the Z boson and the sterile neutrinos; if the sterile neutrinos are light enough, additional invisible Z-decay channels exist [40, 41]. When a heavy sterile neutrino with mass $m_N < m_Z$ mixes with the SM neutrinos, $\nu_i \simeq \nu_\alpha + \theta_s N$ with $\theta_s \ll 1$, one has

$$\Gamma(Z \to \nu N) = \Gamma(Z \to 2\nu) \, |\theta_s|^2 \, \left[1 - \frac{m_N^2}{m_Z^2}\right]^2 \left[1 + \frac{m_N^2}{2m_Z^2}\right]. \tag{38}$$

This result generalizes in an obvious way when a tower of KK neutrinos mixes with the SM:

$$\Gamma(Z \to \nu N_{\pm n}) = \Gamma(Z \to 2\nu) \, |\theta_{\pm n}|^2 \, \left[1 - \frac{M_{\pm n}^2}{m_Z^2}\right]^2 \left[1 + \frac{M_{\pm n}^2}{2m_Z^2}\right]. \tag{39}$$

Note that $\Gamma(Z \to N_{n'} N_n) \ll \Gamma(Z \to \nu N_n)$ for $|\theta_n|^2 \ll 1$ and one need only consider single production. The total non-standard contribution to the invisible width is thus

$$\Gamma(Z \to \nu + \text{invisible}) = 9 \times \sum_n \Gamma(Z \to \nu N_{\pm n}), \tag{40}$$

where three families of active and sterile neutrinos are included, and we again ignore the mixing of the three generations. Neglecting the final state KK neutrino mass in (39), this is approximately

$$\Gamma(Z \to \nu + \text{invisible}) \simeq 9 \, \Gamma(Z \to 2\nu) \times \left[|\theta_0|^2 + \sum_{n>0} \left(|\theta_{+n}|^2 + |\theta_{-n}|^2\right)\right]. \tag{41}$$

The invisible Z-width has been precisely measured as $\Gamma(Z \to \text{invisible}) = 499.0 \pm 1.5$ MeV [42]. This should be compared to the SM prediction of 501.65 ± 0.11 MeV for Z decays to neutrinos. New contributions are severely constrained and should essentially lie within the experimental uncertainty, the 2σ value for which is of order MeV. We plot the invisible Z-width as a function of the bulk mass parameter c in Figure 4. The SM neutrino mass is fixed at $m_\nu = 0.1$ eV and we take $R^{-1} = 1$ GeV, $k = 1.5$ TeV and $k/M_* = 1/2$, while varying λ_N. One observes that the invisible width is well below the $O(1)$ MeV experimental bound for the entire region of parameter space.

6　Other Bounds

Analysis of the bounds on a single sterile-neutrino with $\mathcal{O}(\text{GeV})$ mass can be found in [43] and, more recently, in [44, 45]. The lightest (zero mode) neutrinos in the present model must satisfy these standard bounds. The most robust constraints on an order 100 MeV sterile neutrino come from peak searches in the leptonic decays of pions and kaons [46, 47]. These constraints assume only that the heavy neutrino mixes with the SM and are therefore
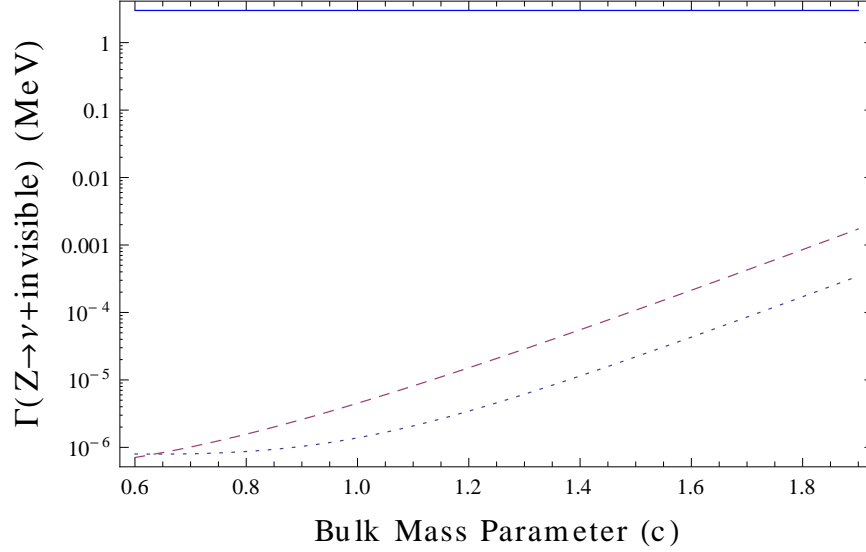
Figure 4: The Invisible Z-Width as a function of the Bulk Mass Parameter (c) for fixed values of the SM neutrino masses, $m_\nu = 0.1$ eV. The dashed (dotted) line is for $\lambda_{\rm N} = 0.5\ (0.1)$ and the solid line at the top shows the upper bound. We use $R^{-1} = 1$ GeV, $k = 1.5$ TeV and $k/M_* = 1/2$ for the plot.

insensitive to the decay properties of the neutrinos. This differs from most collider bounds, which typically make assumptions about the neutrino decay properties to define the signal. For $R^{-1} \sim$ GeV only the zero mode neutrinos can appear in the pion/kaon decays and one may use the data to constrain $|\theta_0|^2$, which may itself be cast as:

$$|\theta_0|^2 \sim \frac{10^{-9}}{3} \times \left(\frac{m_\nu}{0.1\ \mathrm{eV}}\right) \left(\frac{\mathrm{GeV}}{R^{-1}}\right) \left(\frac{10^{-1}}{|\lambda_{\rm N}|}\right), \tag{42}$$

One observes that the constraints of $|\theta_0|^2 \lesssim 10^{-6}$ obtained from kaon decays and $|\theta_0|^2 \lesssim 10^{-5} - 10^{-8}$ from pion decays (see Figure 3 in Ref. [45]) are not severe.

Ref. [48] has detailed a number of bounds for sterile neutrinos that mix with SM neutrinos. Their focus was primarily on neutrinos with mass $M_N > m_Z/2$ but some of their bounds are reliable for lighter masses. Noting that mixing with sterile neutrinos in general introduces a correction to the muon decay constant they derive bounds from lepton universality, CKM unitarity[10] and the invisible Z-decay width. For $M_N > m_Z/2$ the latter is affected by the (slightly) modified coupling of SM neutrinos to the Z; in this work we have KK neutrinos satisfying $M_N < m_Z/2$ and the invisible Z width is modified directly by the additional decay channels $Z \to \nu N_n$ as detailed in Section 5.

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[10]The observed CKM mixing elements are extracted with some sensitivity to the muon decay constant; a shift in the latter modifies the former.

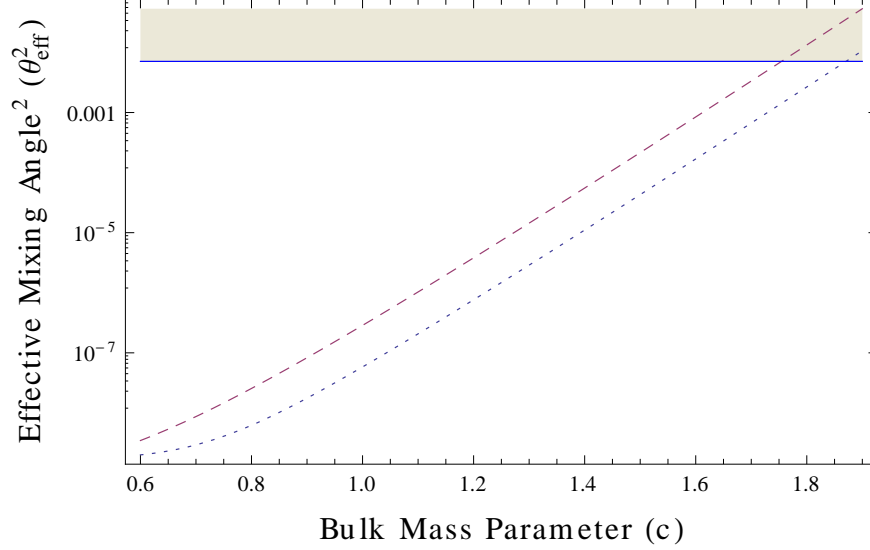Figure 5: The Effective Active-Sterile Mixing-Angle Squared $\left(\sum_n \theta_n^2\right)$ as a function of the Bulk Mass Parameter (c) for fixed values of the SM neutrino masses, $m_\nu = 0.1$ eV. The dashed (dotted) line is for $\lambda_N = 0.5$ (0.1) and the shaded region is excluded. We use $R^{-1} = 1$ GeV, $k = 1.5$ TeV and $k/M_* = 1/2$ for the plot.

The bounds of Ref. [48] can be summarized as

$$
\begin{aligned}
\theta_e^2 &< 7.1 \times 10^{-3} \quad \text{for} \quad M_N > 0.14 \text{ GeV } (m_\pi), \\
\theta_\mu^2 &< 1.4 \times 10^{-3} \quad \text{for} \quad M_N > 1.15 \text{ GeV } (m_\Lambda), \\
\theta_\tau^2 &< 1.7 \times 10^{-2} \quad \text{for} \quad M_N > 1.777 \text{ GeV } (m_\tau).
\end{aligned} \tag{43}
$$

Here θ_α^2 is an effective mixing angle between the SM neutrino with lepton flavour α and sterile neutrinos with masses exceeding the quoted values. In the present context the effective angle is defined as $\theta_{\text{eff}}^2/3 = \theta_0^2 + \sum_n(\theta_{-n}^2 + \theta_{+n}^2)$, with the factor of three accounting for three generations of bulk neutrinos. For simplicity we do not differentiate the different generations in this work and assume that all mixing angles are of the same order. In this case a conservative bound of

$$
\theta_{\text{eff}}^2 = 3 \times \left\{\theta_0^2 + \sum_{n>0}(\theta_{-n}^2 + \theta_{+n}^2)\right\} < 7.1 \times 10^{-3} \quad \text{for} \quad M_0 > m_\pi, \tag{44}
$$

is obtained. We plot the effective active-sterile mixing-angle squared as a function of the bulk mass parameter (c) in Figure 5, along with the experimental bound. The figure shows that values of $c \lesssim 1.85$ (1.7) are consistent with the bounds for $\lambda_N = 0.1$ (0.5).

Light sterile neutrinos can also be created in collider experiments. Kinematics permitting, sterile neutrinos can be singly produced on the Z peak and can decay via an off-shell gauge boson, $N \to \nu + Z^*$ with $Z^* \to \ell^+\ell^-$, $\bar{q}q$, or $N \to \ell + W^*$ with $W^* \to \ell\nu_\ell$, qq'. These decay signals have been sought at LEP for gauge-singlet neutrinos in the mass range 3 GeV <

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$M_N < m_Z$ [49]. The resulting bound is essentially constant at $|\theta_s|^2 \lesssim 2 \times 10^{-4}$ for M_N in the range 3 GeV $< M_N < 50$ GeV and decreases slowly (rapidly) for lighter (heavier) M_N; see Figures 6 and 7 in Ref. [49].

In the present framework, each member of the KK tower can decay to SM fields via off-shell gauge bosons[11] and it appears that the LEP bounds should be applied to all of the KK neutrinos. The LEP bounds certainly apply to the zero mode neutrinos, which decay through their inherited SM couplings and behave essentially like a standard, light, sterile neutrino. One thus requires $|\theta_0|^2 \lesssim 10^{-4}$ for $(M_0/\mathrm{GeV}) \in [3, 50]$, with the bound dropping to $|\theta_0|^2 \lesssim 10^{-2}$ as M_0 decreases to 500 MeV. Given that $\theta_0^2 \simeq 10^{-10} \times (m_\nu/10^{-1}\mathrm{eV}) \times (\mathrm{GeV}/|M_{00}|)$ this bound is not restrictive.

The higher modes $(n > 0)$ can also decay to the SM. However, one should not apply the bounds of Ref. [49] directly to these modes. These bounds are obtained under the assumption that the sterile neutrinos *only* decay to Standard Model final states. This assumption does not hold for the $n > 0$ KK modes, which can decay within the hidden sector via KK graviton production, $N_n \to N_m + h_a$, where h_a is the ath KK graviton. The KK neutrinos and gravitons are both localized towards the IR brane and in general have large wavefunction overlaps. Despite the vertex being gravitational in nature, the relevant coupling strength is of order $R \gg M_{Pl}^{-1}$ and the hidden sector decays cannot be neglected. Graviton decays are allowed for $n > m + a$ and the partial widths go like $\Gamma_n \sim (k/M_*)^3 M_n^3 R^2$. The modes are increasingly broad as one goes up the KK tower and, for a given value of n, decays with $n \sim a + m$ are preferred. The total two-body hidden width for the nth mode is thus of order[12] $n \times \Gamma_n$. Decay of the nth mode to the SM is suppressed by a factor of $|\theta_{\pm n}|^2 \ll 1$ so that, absent hierarchically small values of k/M_*, decays within the hidden sector will dominate.

Once a given KK neutrino decays into the hidden sector, the daughter neutrino(s) and graviton(s) will further decay within the hidden sector (when kinematically allowed). The graviton can decay via $h_a \to N_m N_n$ with width $\Gamma_a \sim (k/M_*)^3 m_a^3 R^2$ (similar to RS [50]) and shows a preference for decays with $a \sim m + n$. Thus a hidden sector "shower" occurs, consisting of a series of cascade decays down the hidden KK towers until one has a collection of light KK modes with mass of order R^{-1}, which in turn decay back to the SM. The expected signal from single production of a higher KK neutrino is therefore very different to the signal searched for at LEP [49]. Rather than applying the constraint $|\theta_s|^2 \lesssim 10^{-4}$ directly to $|\theta_{\pm n}|$ one should instead apply it to the quantity $BR(N_{\pm n} \to \mathrm{SM}) \times |\theta_{\pm n}|^2$. Given that $BR(N_{\pm n} \to \mathrm{SM}) \ll 1$ and $|\theta_{\pm n}|^2 < 10^{-4}$ for the parameters of interest here $(k \lesssim M_* \sim \mathrm{TeV}, R^{-1} \sim \mathrm{GeV}$ and $c \simeq 1)$, the LEP bound does not impose an additional constraint. Bounds resulting from heavy neutrino searches at beam dump experiments by, e.g., the WA66 Collaboration [51], are similarly unrestrictive in the present context.

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[11]Modes with mass $M_n > m_Z$ can also decay directly to on-shell vectors.

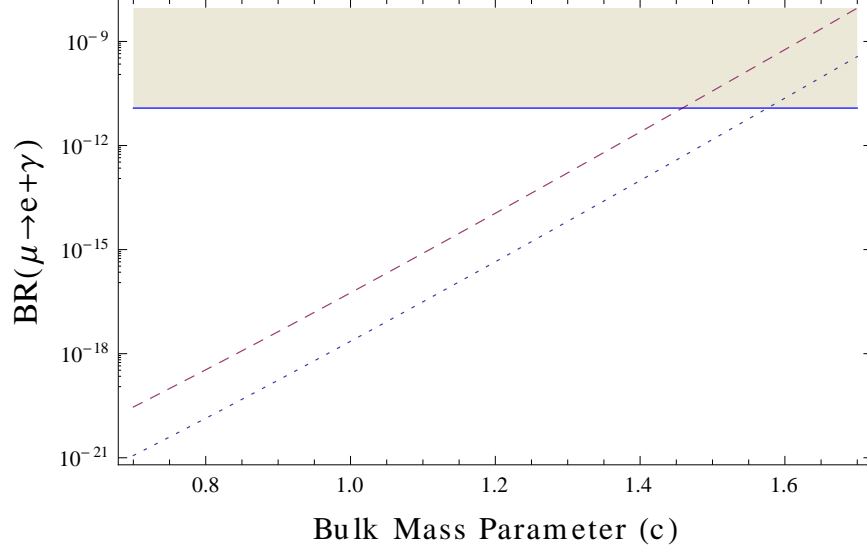[12]This behaviour is similar to the hidden KK vector widths detailed in Ref. [20].

Figure 6: The $\mu \to e + \gamma$ Branching Ratio as a function of the Bulk Mass Parameter (c) for fixed values of the SM neutrino masses, $m_\nu = 0.1$ eV. The dashed (dotted) line is for $\lambda_N = 0.5$ (0.1) and the shaded region is experimentally excluded. We use $R^{-1} = 1$ GeV, $k = 1.5$ TeV and $k/M_* = 1/2$ for the plot.

7 FCNC Bounds: $\mu \to e + \gamma$

Indirect constraints on the parameter space for sterile neutrinos can be derived from searches for flavour changing neutral current processes like $\mu \to e + e^+ e^-$, $\mu \to e + \gamma$ and $\mu - e$ conversion in nuclei [52]. For sterile neutrinos mixing with the SM, the induced $\mu \to e + \gamma$ branching fraction can be written as [52, 53, 54]:

$$BR(\mu \to e + \gamma) = \frac{3\alpha}{8\pi} \left| \sum_n \theta_{en} \, \theta_{\mu n}^* \, \mathcal{F}(M_n^2/m_W^2) \right|^2, \tag{45}$$

where the function $\mathcal{F}(x)$ is

$$\mathcal{F}(x) = x[1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln(x)]/2(1-x)^4, \tag{46}$$

with $\mathcal{F}(x) \in [0, 1]$ for $x \in [0, \infty)$. Present bounds require $BR(\mu \to e + \gamma) < 1.2 \times 10^{-11}$ at the 90% C.L. [55].

We plot the $\mu \to e + \gamma$ branching ratio as a function of the bulk mass parameter c in Figure 6 for three generations of bulk neutrinos with the sum again cut off at $n_* \simeq (M_* R)/\pi$. The SM neutrino masses are fixed at $m_\nu = 0.1$ eV and we use $k = 1.5$ TeV, $R^{-1} = 1$ GeV and $k/M_* = 1/2$ for the plot. The figure shows that $\mu \to e + \gamma$ constraints are more stringent than the constraints previously considered; values of $c \gtrsim 1.6$ (1.45) are already excluded for $\lambda_N = 0.1$ (0.5). Recall that $\lambda \in [10^{-2}, 0.1]$ when $c \in [1.3, 1.6]$ for $\lambda_N = 0.1$ and $m_\nu \simeq 0.1$ eV. Thus the interesting region of parameter space in which the Yukawa coupling λ is $\mathcal{O}(0.1)$ corresponds to values of c that are close to the upper bound.

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Other flavour changing processes like $\mu - e$ conversion in nuclei and $\mu \to e + e^+ e^-$ give similar constraints. The most accurate experimental result in this regard comes from searches for $\mu - e$ conversion in Ti. The branching ratio with respect to the total nuclear muon capture rate is bound as $BR(\mu \text{ Ti} \to e \text{ Ti}) < 4.3 \times 10^{-12}$ [56]. However, this bound turns out to be less severe than the $\mu \to e + \gamma$ constraint. For a single sterile neutrino, the resulting constraint is $\left| \theta_{es} \theta_{\mu s}^* \sum_\ell |\theta_{\ell s}|^2 \right| < 1.3 \times 10^{-3} (100 \text{ GeV}/M_N)^2$, which is weaker due to the $\sim \theta_{\alpha s}^4$ dependence unless $M_N \gg 10$ TeV. We only consider $M_n \lesssim 1$ TeV in this work and thus find the $\mu \to e + \gamma$ bound more useful.

In summary, we find that $\mu \to e + \gamma$ constraints on the present model are the most severe and that these typically require $c \lesssim 1.6$ for $\lambda_{\scriptscriptstyle N} \simeq 0.1$. Of interest is the fact that larger values of c correspond to larger values of the Yukawa coupling λ for a given value of m_ν (see Figure 2). The more interesting regions of parameter space thus correspond to values of $BR(\mu \to e + \gamma)$ just below current limits. In particular, Yukawa couplings of $\lambda \gtrsim 0.01$ correspond to $BR(\mu \to e + \gamma) \gtrsim 10^{-14}$. The MEG collaboration is expected to report new results on the branching fraction for $\mu \to e + \gamma$ soon [11]. Given their targeted precision of measuring $BR(\mu \to e + \gamma)$ at the $\mathcal{O}(10^{-13})$ level, MEG will have the potential to significantly improve the constraints on the present model. In existing releases the MEG collaboration has reported that the best value for the number of signal events in their likelihood analysis is nonzero (specifically, three events) [11]. Interpreted as a signal, this corresponds to $BR(\mu \to e + \gamma) = 3 \times 10^{-12}$, which is right in the interesting range in the present context (see Figure 6). We eagerly await the release of more data by the MEG collaboration to see if this potential signal holds up.

8   Hidden Higgs Decays

The KK tower of sterile neutrinos has the potential to modify the Higgs decay widths due to additional channels involving the sterile states. The Higgs couples to the steriles via the UV localized Yukawa interaction

$$S \ \supset \ -\frac{\lambda}{\sqrt{M_*}} \int d^5 x \sqrt{-g_{uv}} \ \bar{L} H N_R \ \delta(z - k^{-1}) \,, \tag{47}$$

where L is a lepton doublet and H is the SM scalar doublet. In a general 4D theory, the SM Higgs will decay into neutrino pairs $h \to \nu N$ whenever the SM neutrinos Yukawa-couple to one or more light sterile neutrinos. However, the demand $m_\nu \lesssim \mathcal{O}(\text{eV})$ typically forces the Yukawa couplings to be tiny as one expects $\lambda_s \simeq \sqrt{m_\nu M_N}/\langle H \rangle$ for seesaw neutrino masses; $M_N \sim$ GeV gives $\lambda_s \simeq 10^{-7}$ for $m_\nu \simeq 0.1$ eV and the corresponding modification to the Higgs width is therefore negligible.

In the present context, the Yukawa couplings need not be tiny as m_ν is suppressed by warping (or in the dual 4D theory, fundamental/composite mixing) in addition to the usual

seesaw suppression. The action can be written as

$$S \supset -\frac{\lambda}{\sqrt{M_*}} \int d^5x \sqrt{-g_{uv}} \ \bar{L}HN_R \ \delta(z - k^{-1})$$

$$= -\sum_n \int d^4x \ \frac{m_n^{\mathrm{D}}}{\langle H \rangle} \ h\bar{\nu}_L \nu_R^{(n)} \ + \ \ldots$$

$$= -\int d^4x \ \ h\bar{\nu}_L \left\{ \lambda_0^{\mathrm{eff}} N_{0R} + \sum_{n>0} \lambda_n^{\mathrm{eff}} \left(N_{+nL}^c + N_{-nL}^c \right) \right\} \ + \ \ldots$$

where we define $\lambda_0^{\mathrm{eff}} \simeq \sqrt{m_\nu M_{00}}/\langle H \rangle$, use the fact that $\nu_R^{(n)c} \simeq (N_{+n} + N_{-n})_L/\sqrt{2}$, and write the effective Yukawa coupling for the nth KK mode as

$$\lambda_n^{\mathrm{eff}} \equiv \frac{m_n^{\mathrm{D}}}{\sqrt{2}\langle H \rangle} \simeq 2\sqrt{|\lambda_{\mathrm{N}}|} \times \frac{10^{-5}}{\langle H \rangle R} \left(\frac{m_\nu}{10^{-1} \ \mathrm{eV}} \right)^{1/2} \left(\frac{\mathrm{GeV}}{R^{-1}} \right)^{1/2} \left(\frac{M_n R}{2} \right)^c. \tag{48}$$

If one adds a sterile neutrino N with Yukawa coupling λ_s to the SM, the decay width for the Higgs to two neutrinos is:

$$\Gamma_N(h \to \nu \ N) \ = \ \frac{\lambda_s^2 m_h}{8\pi} \left(1 - \frac{M_N^2}{m_h^2} \right)^2 \tag{49}$$

where we neglect $m_\nu \ll M_N$. Neglecting the phase space factors, we can apply this result to the present model:

$$\Gamma(h \to \mathrm{invisible}) \equiv 9 \times \sum_n \Gamma(h \to \nu \ N_n) \simeq \frac{9m_h}{8\pi} \times \left\{ (\lambda_0^{\mathrm{eff}})^2 + \sum_{n>0} 2(\lambda_n^{\mathrm{eff}})^2 \right\}, \tag{50}$$

where the sum is cut off in the UV at $n_h \simeq m_h R/\pi$.

This invisible width is plotted as a function of the bulk mass parameter c in Figure 7. Observe that it is significantly larger than that obtained in a standard low-scale seesaw with order GeV sterile neutrinos (e.g., the νMSM), for which one has $\Gamma_N \sim (\lambda_s^2/8\pi)m_h \sim 10^{-12}$ GeV for $m_h \sim 100$ GeV. Noting that the width for SM Higgs decays is $\Gamma_{\mathrm{SM}} \sim \mathcal{O}(10^{-3}) \ [\mathcal{O}(100)]$ GeV for $m_h = 150$ GeV [600 GeV], the invisible branching fraction is $\Gamma_{\mathrm{inv}}/\Gamma_{\mathrm{SM}} \lesssim 10^{-3} \ [10^{-4}]$ for the values shown in the figure. For a lighter Higgs mass with larger values of c the invisible branching fraction is on the order of the two-photon width, $BR_{\mathrm{inv}} \sim BR(h \to 2\gamma)$. Despite its small width, the two-photon channel is known to be useful if the Higgs is light, due to the relatively clean signal. However, invisible decays are not as readily discerned from the data. The LHC can "observe" invisible Higgs decays, but only for much larger branching fractions of roughly 13% (5%) for 10 fb^{-1} (100 fb^{-1}) of data [57]. We conclude that, despite a significant increase relative to other models with a low-scale seesaw, the width for Higgs decays to neutrinos remains too small to be observed at the LHC. The ILC would have a better discovery (and limits) reach for invisible Higgs decays, possibly achieving discovery (limits) for invisible branching fractions of order 1% (0.1%) [58].
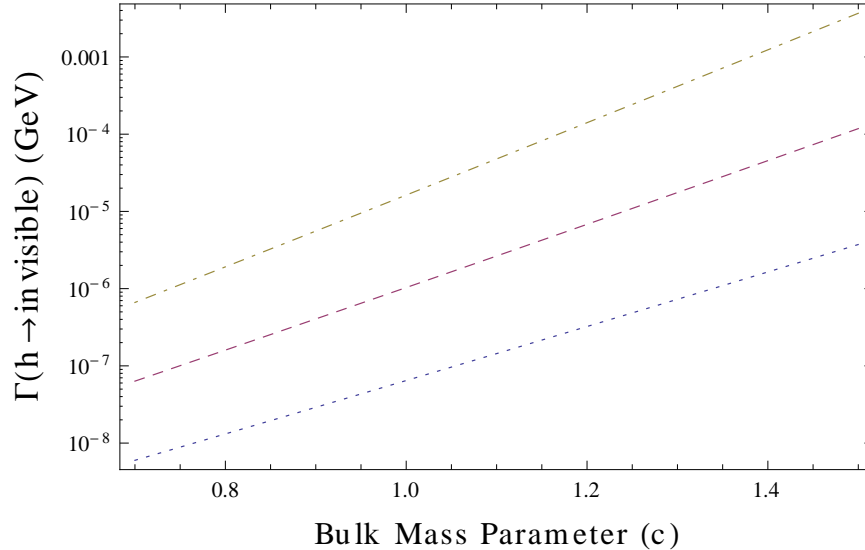
Figure 7: The Invisible Higgs Decay Width as a function of the Bulk Mass Parameter (c) for fixed values of the SM neutrino masses, $m_\nu = 0.1$ eV. The dotted (dashed, dot-dashed) line is for $m_h = 150$ (300, 600) GeV with $\lambda_{\mathrm{N}} = 0.5$ and $R^{-1} = 1$ GeV.

We have assumed "natural values" for the mixing angles in this analysis; e.g., the natural value for the mixing angle between a standard seesaw pair ν_L and N_s is $\theta_s \simeq m_\nu / M_N$. However, the mixing between the SM neutrinos and the heavy neutrinos need not be as small as this natural value if there are cancellations amongst the parameters in the seesaw formula [24]. An enhanced mixing angle means the Dirac Yukawa-coupling can be larger, leading to an enhancement of the hidden Higgs decay width [24, 59]. Such cancellations could be considered in the present context and the hidden Higgs width can be similarly enhanced.

We also note that the total invisible width can potentially be higher than the neutrino width. Stabilization of the length of the extra dimension via the Goldberger-Wise mechanism, for example, could open additional invisible channels. The Higgs can couple to, and possibly mix with, the Goldberger-Wise scalar and thus the overall invisible width could increase. This is beyond the scope of the present work but we hope to consider it in the future.

9   Conclusion

The mini-seesaw mechanism achieves light SM neutrino masses by combining naturally suppressed Dirac and (sterile) Majorana mass-scales together in a low-scale seesaw mechanism. It employs a truncated ("little") warped space that, via AdS/CFT, is dual to a 4D theory possessing conformal symmetry in some window, $M_* > E > R^{-1}$, with $M_* \ll M_{Pl}$. The model generates light SM neutrino masses without recourse to a large UV scale, and thus

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offers hope that the mechanism of neutrino mass generation may be explored in feasible experiments.

A key feature of this approach is the existence of a tower of light sterile-neutrinos that mix with the SM. We have considered the detailed bounds on these light neutrinos from processes like $0\nu\beta\beta$-decay, $\mu \to e + \gamma$ and invisible Z-decays. We find that the most stringent constraints come from FCNC processes like $\mu \to e + \gamma$. Nonetheless we have shown that viable parameter space exists in which the input Yukawa couplings can be sizable. The model therefore provides a viable explanation for the lightness of the known SM neutrinos. Furthermore, the $\mu \to e + \gamma$ branching fraction lies just below current experimental bounds for interesting regions of parameter space and, importantly, is within the region to be probed by the forthcoming MEG experiment. We look forward to the MEG results in the hope that (indirect) evidence for the origin of neutrino mass is revealed.

Acknowledgements

A   Appendix: Mass Mixing

To $\mathcal{O}(M_{nn'}/M_n)$ the matrix that gives the heavy eigenstates is:

$$
U_{KK} = \begin{pmatrix}
1 & -\frac{1}{\sqrt{2}}(M_{01}/M_1) & \frac{1}{\sqrt{2}}(M_{01}/M_1) & -\frac{1}{\sqrt{2}}(M_{02}/M_2) & \frac{1}{\sqrt{2}}(M_{02}/M_2) & \cdots \\
0 & \frac{1}{\sqrt{2}}(1 - \frac{M_{11}}{4M_1}) & \frac{1}{\sqrt{2}}(1 + \frac{M_{11}}{4M_1}) & -\frac{M_{12}}{\sqrt{2}}\frac{M_2}{M_2^2 - M_1^2} & \frac{M_{12}}{\sqrt{2}}\frac{M_2}{M_2^2 - M_1^2} & \cdots \\
M_{01}/M_1 & \frac{1}{\sqrt{2}}(1 + \frac{M_{11}}{4M_1}) & \frac{1}{\sqrt{2}}(-1 + \frac{M_{11}}{4M_1}) & -\frac{M_{12}}{\sqrt{2}}\frac{M_1}{M_2^2 - M_1^2} & -\frac{M_{12}}{\sqrt{2}}\frac{M_1}{M_2^2 - M_1^2} & \cdots \\
0 & -\frac{M_{12}}{\sqrt{2}}\frac{M_1}{M_1^2 - M_2^2} & \frac{M_{12}}{\sqrt{2}}\frac{M_1}{M_1^2 - M_2^2} & \frac{1}{\sqrt{2}}(1 - \frac{M_{22}}{4M_2}) & \frac{1}{\sqrt{2}}(1 + \frac{M_{22}}{4M_2}) & \cdots \\
M_{02}/M_2 & -\frac{M_{12}}{\sqrt{2}}\frac{M_2}{M_1^2 - M_2^2} & -\frac{M_{12}}{\sqrt{2}}\frac{M_2}{M_1^2 - M_2^2} & \frac{1}{\sqrt{2}}(1 + \frac{M_{22}}{4M_2}) & \frac{1}{\sqrt{2}}(-1 + \frac{M_{22}}{4M_2}) & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}.
$$

References

[1] J. W. F. Valle, arXiv:1001.5189 [hep-ph]; A. Strumia and F. Vissani, arXiv:hep-ph/0606054; R. N. Mohapatra and A. Y. Smirnov, Ann. Rev. Nucl. Part. Sci. **56**, 569 (2006) [arXiv:hep-ph/0603118]; R. N. Mohapatra *et al.*, Rept. Prog. Phys. **70**, 1757 (2007) [arXiv:hep-ph/0510213]; G. Altarelli and F. Feruglio, New J. Phys. **6**, 106 (2004) [arXiv:hep-ph/0405048]; S. F. King, Rept. Prog. Phys. **67**, 107 (2004) [arXiv:hep-ph/0310204].

[2] T. Schwetz, M. Tortola, J. W. F. Valle, arXiv:1103.0734 [hep-ph].

[3] T. Yanagida, in Proceedings of the *"Workshop on the Unified Theory and the Baryon Number in the Universe"*, Tsukuba, Japan, Feb. 13-14, 1979, edited by O. Sawada and A. Sugamoto, KEK report KEK-79-18, p. 95, and *"Horizontal Symmetry And Masses Of Neutrinos"*, Prog. Theor. Phys. **64** (1980) 1103; M. Gell-Mann, P. Ramond and R. Slansky, in *"Supergravity"* (North-Holland, Amsterdam, 1979) *eds.* D. Z. Freedom and P. van Nieuwenhuizen, Print-80-0576 (CERN); P. Minkowski, Phys. Lett. B **67**, 421 (1977); S. L. Glashow, *The future of elementary particle physics*, in *Proceedings of the 1979 Cargèse Summer Institute on Quarks and Leptons* (M. Lévy et al. eds.), Plenum Press, New York, 1980, p. 687; R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).

[4] A. Zee, Nucl. Phys. **B264**, 99 (1986); K. S. Babu, Phys. Lett. **B203**, 132 (1988).

[5] K. A. Meissner, H. Nicolai, Phys. Lett. **B648**, 312-317 (2007) [arXiv:hep-th/0612165]; R. Foot at al, Phys. Rev. **D76**, 075014 (2007) [arXiv:0706.1829 [hep-ph]]; R. Foot et al, Phys. Rev. **D77**, 035006 (2008) [arXiv:0709.2750 [hep-ph]]; S. Iso, N. Okada, Y. Orikasa, Phys. Lett. **B676**, 81-87 (2009) [arXiv:0902.4050 [hep-ph]].

[6] N. Arkani-Hamed, S. Dimopoulos, G. R. Dvali, J. March-Russell, Phys. Rev. **D65**, 024032 (2002) [arXiv:hep-ph/9811448]; K. R. Dienes, E. Dudas, T. Gherghetta, Nucl. Phys. **B557**, 25 (1999) [arXiv:hep-ph/9811428].

[7] N. Arkani-Hamed and Y. Grossman, Phys. Lett. B **459**, 179 (1999) [arXiv:hep-ph/9806223]; T. Okui, JHEP **0509**, 017 (2005) [arXiv:hep-ph/0405083]; Y. Grossman and Y. Tsai, JHEP **0812**, 016 (2008) [arXiv:0811.0871 [hep-ph]]; Y. Grossman, D. J. Robinson, JHEP **1101**, 132 (2011) [arXiv:1009.2781 [hep-ph]]; R. S. Hundi, S. Roy, arXiv:1105.0291 [hep-ph].

[8] K. L. McDonald, Phys. Lett. **B696**, 266-272 (2011) [arXiv:1010.2659 [hep-ph]].

[9] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998) [Int. J. Theor. Phys. **38**, 1113 (1999)] [arXiv:hep-th/9711200]; S. S. Gubser, I. R. Klebanov and A. M. Polyakov, Phys. Lett. B **428**, 105 (1998) [arXiv:hep-th/9802109]; E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998) [arXiv:hep-th/9802150].

[10] N. Arkani-Hamed, M. Porrati and L. Randall, JHEP **0108** (2001) 017 [arXiv:hep-th/0012148]; R. Rattazzi and A. Zaffaroni, JHEP **0104**, 021 (2001) [arXiv:hep-th/0012248].

[11] R. Sawada, http://pos.sissa.it/archive/conferences/120/263/ICHEP%202010_263.pdf; G. Cavoto, arXiv:1012.2110 [hep-ex].

[12] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999) [arXiv:hep-ph/9905221].

[13] Y. Grossman and M. Neubert, Phys. Lett. B **474**, 361 (2000) [arXiv:hep-ph/9912408].

[14] T. Gherghetta and A. Pomarol, Nucl. Phys. B **586**, 141 (2000) [arXiv:hep-ph/0003129]; S. J. Huber and Q. Shafi, Phys. Lett. B **498**, 256 (2001) [arXiv:hep-ph/0010195].

[15] S. J. Huber and Q. Shafi, Phys. Lett. B **544**, 295 (2002) [arXiv:hep-ph/0205327]; S. J. Huber and Q. Shafi, Phys. Lett. B **583**, 293 (2004) [arXiv:hep-ph/0309252].

[16] T. Gherghetta, Phys. Rev. Lett. **92**, 161601 (2004) [arXiv:hep-ph/0312392].

[17] B. Gripaios, Nucl. Phys. B **768**, 157 (2007) [Erratum-ibid. **830**, 390 (2010)] [arXiv:hep-ph/0611218]; T. Gherghetta, K. Kadota and M. Yamaguchi, Phys. Rev. D **76**, 023516 (2007) [arXiv:0705.1749 [hep-ph]]; G. Perez and L. Randall, JHEP **0901** (2009) 077 [arXiv:0805.4652 [hep-ph]]; C. Csaki, C. Delaunay, C. Grojean and Y. Grossman, JHEP **0810** (2008) 055 [arXiv:0806.0356 [hep-ph]]; F. del Aguila, A. Carmona and J. Santiago, JHEP **1008** (2010) 127 [arXiv:1001.5151 [hep-ph]]; A. Kadosh and E. Pallante, JHEP **1008**, 115 (2010) [arXiv:1004.0321 [hep-ph]]; A. Watanabe and K. Yoshioka, arXiv:1007.1527 [hep-ph].

[18] G. von Gersdorff and M. Quiros, Phys. Lett. B **678**, 317 (2009) [arXiv:0901.0006 [hep-ph]].

[19] T. Flacke and D. Maybury, JHEP **0703** (2007) 007 [arXiv:hep-ph/0612126]; K. L. McDonald and D. E. Morrissey, JHEP **1005**, 056 (2010) [arXiv:1002.3361 [hep-ph]].

[20] K. L. McDonald, D. E. Morrissey, JHEP **1102**, 087 (2011) [arXiv:1010.5999 [hep-ph]].

[21] H. Pas, S. Pakvasa, T. J. Weiler, Phys. Rev. **D72**, 095017 (2005) [arXiv:hep-ph/0504096].

[22] T. Asaka, S. Blanchet and M. Shaposhnikov, Phys. Lett. B **631** (2005) 151 [arXiv:hep-ph/0503065]; T. Asaka and M. Shaposhnikov, Phys. Lett. B **620**, 17 (2005) [arXiv:hep-ph/0505013]; D. Gorbunov, M. Shaposhnikov, JHEP **0710**, 015 (2007) [arXiv:0705.1729 [hep-ph]]

[23] T. Asaka, S. Eijima, H. Ishida, JHEP **1104**, 011 (2011) [arXiv:1101.1382 [hep-ph]].

[24] A. de Gouvea, arXiv:0706.1732 [hep-ph].

[25] A. Kusenko, Phys. Rept. **481**, 1 (2009) [arXiv:0906.2968 [hep-ph]].

[26] A. Ioannisian, A. Pilaftsis, Phys. Rev. **D62**, 066001 (2000) [arXiv:hep-ph/9907522].

[27] T. Fukuyama, N. Okada, arXiv:1104.1736 [hep-ph].

[28] A. Ibarra, C. Simonetto, JHEP **0804**, 102 (2008) [arXiv:0802.3858 [hep-ph]]; A. Abada, C. Biggio, F. Bonnet, M. B. Gavela, T. Hambye, Phys. Rev. **D78**, 033007 (2008) [arXiv:0803.0481 [hep-ph]]; D. Suematsu, T. Toma, T. Yoshida, Phys. Rev. **D79**, 093004 (2009) [arXiv:0903.0287 [hep-ph]]; J. Hisano, M. Nagai, P. Paradisi, Y. Shimizu, JHEP **0912**, 030 (2009) [arXiv:0904.2080 [hep-ph]]; S. Blanchet, T. Hambye, F. - X. Josse-Michaux, JHEP **1004**, 023 (2010) [arXiv:0912.3153 [hep-ph]]; M. Davidkov, D. I. Kazakov, arXiv:1102.1582 [hep-ph].

[29] W. D. Goldberger and M. B. Wise, Phys. Rev. Lett. **83**, 4922 (1999) [arXiv:hep-ph/9907447]; J. Garriga and A. Pomarol, Phys. Lett. B **560**, 91 (2003) [arXiv:hep-th/0212227].

[30] H. Davoudiasl, G. Perez and A. Soni, Phys. Lett. B **665**, 67 (2008) [arXiv:0802.0203 [hep-ph]]; H. Davoudiasl, T. McElmurry, A. Soni, Phys. Rev. **D82**, 115028 (2010) [arXiv:1009.0764 [hep-ph]].

[31] B. Batell, T. Gherghetta, Phys. Rev. **D76**, 045017 (2007) [arXiv:0706.0890 [hep-th]].

[32] A. Watanabe, K. Yoshioka, Phys. Lett. **B683**, 289-293 (2010) [arXiv:0910.0677 [hep-ph]].

[33] A. Watanabe, Private Communication.

[34] W. D. Goldberger, M. B. Wise, Phys. Lett. **B475**, 275-279 (2000) [arXiv:hep-ph/9911457]; C. Charmousis, R. Gregory, V. A. Rubakov, Phys. Rev. **D62**, 067505 (2000) [arXiv:hep-th/9912160].

[35] M. S. Carena, J. D. Lykken, M. Park, Phys. Rev. **D72**, 084017 (2005) [arXiv:hep-ph/0506305].

[36] W. Rodejohann, arXiv:1011.4942 [hep-ph].

[37] H. V. Klapdor-Kleingrothaus, H. Pas, [arXiv:hep-ph/0002109].

[38] A. Dueck, W. Rodejohann and K. Zuber, arXiv:1103.4152 [hep-ph].

[39] M. Blennow, E. Fernandez-Martinez, J. Lopez-Pavon, J. Menendez, JHEP **1007**, 096 (2010) [arXiv:1005.3240 [hep-ph]].

[40] M. Gronau, C. N. Leung, J. L. Rosner, Phys. Rev. **D29**, 2539 (1984); M. Gronau, J. L. Rosner, Phys. Lett. **B147**, 217 (1984).

[41] M. Dittmar, A. Santamaria, M. C. Gonzalez-Garcia, J. W. F. Valle, Nucl. Phys. **B332**, 1 (1990).

[42] K. Nakamura *et al.* (Particle Data Group), J. Phys. **G37**, 075021 (2010).

[43] A. Y. Smirnov, R. Zukanovich Funchal, Phys. Rev. **D74**, 013001 (2006) [arXiv:hep-ph/0603009].

[44] F. del Aguila, J. de Blas, M. Perez-Victoria, Phys. Rev. **D78**, 013010 (2008) [arXiv:0803.4008 [hep-ph]].

[45] A. Atre, T. Han, S. Pascoli, B. Zhang, JHEP **0905**, 030 (2009) [arXiv:0901.3589 [hep-ph]].

[46] D. I. Britton *et al.*, Phys. Rev. Lett. **68** (1992) 3000; D. I. Britton *et al.*, Phys. Rev. D **46** (1992) 885.

[47] D. Berghofer *et al.*, Proc. Inern. Conf. on Neutrino Physics and Astrophysics (Maui, Hawaii, 1981), 67 (1981), eds. R. J. Cence, E. Ma and A. Roberts, Vol. II (University of Hawaii, Honolulu, HI, 1981); T. Yamazaki *et al.*, Proc. 22nd Intern. Conf. on High Energy Physics (Leipzig, 1984), 262 (1984), eds. A. Meyer and E. Wierczorek, Vol. I (Akademie der Wiessenachaften der DDR, Leipzig, 1984).

[48] E. Nardi, E. Roulet, D. Tommasini, Phys. Lett. **B327**, 319-326 (1994) [arXiv:hep-ph/9402224].

[49] O. Adriani *et al.* [L3 Collaboration], Phys. Lett. **B295**, 371-382 (1992).

[50] H. Davoudiasl, J. L. Hewett, T. G. Rizzo, Phys. Rev. Lett. **84** (2000) 2080 [arXiv:hep-ph/9909255].

[51] A. M. Cooper-Sarkar *et al.* [WA66 Collaboration], Phys. Lett. **B160**, 207 (1985).

[52] D. Tommasini, G. Barenboim, J. Bernabeu, C. Jarlskog, Nucl. Phys. **B444**, 451-467 (1995) [arXiv:hep-ph/9503228].

[53] E. Ma, A. Pramudita, Phys. Rev. **D24**, 1410 (1981).

[54] P. Langacker, D. London, Phys. Rev. **D38**, 886 (1988).

[55] M. L. Brooks *et al.* [MEGA Collaboration], Phys. Rev. Lett. **83**, 1521-1524 (1999) [arXiv:hep-ex/9905013].

[56] C. Dohmen *et al.* [SINDRUM II. Collaboration], Phys. Lett. **B317**, 631-636 (1993).

[57] O. J. P. Eboli, D. Zeppenfeld, Phys. Lett. **B495**, 147-154 (2000) [arXiv:hep-ph/0009158].

[58] F. Richard, P. Bambade, arXiv:hep-ph/0703173.

[59] J. -H. Chen, X. -G. He, J. Tandean, L. -H. Tsai, Phys. Rev. **D81**, 113004 (2010) [arXiv:1001.5215 [hep-ph]].